Control Systems Project and Seminar

Sub: Simulation of Flight Control System of F12 Aircraft

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Introduction:

The F-12 series of aircraft, commonly referred to as the “Blackbirds”, were designed in early sixties in US. Here the dynamics and the major control surfaces of F-12 have been studied to formulate the force, kinematic, moment and navigational equations. A longitudinal model is defined for the air-profile and the engine to simulate the dynamics of F-12. The simulation is performed in MATLAB environment and the test results and plots are presented and analyzed.

Crude Idea of Dynamics:

The flight of the aircraft is realized by the followings:

- Lift generated by the flow of air over the wings
- Thrust generated by the propellers and jets to balance the drag force.
- Structure and control surfaces must be strong enough without being too heavy.
- The aircraft can be trimmed or balanced(i.e. moment about cg=0).
- The entire system should be either stable or controllable.

Aerodynamic Control Surfaces:

- Elevators to control Pitch Angle.
- Ailerons to control Roll angle.
- Rudders to control Yaw.
- Flaps to increase lift and drag.
- Leading edge slats to increase lift.

Spoilers to reduce lift and increase drag.

Dynamic and Kinematic Analysis:

The dynamics has two part:

1. Translational Motion
2. Rotational Motion

Few notations which are used in following equations to describe dynamics and kinematics :

- VT = Translational Velocity
- β = Side-slip angle
- α = Angle of attack

- Ψ = Yaw angle= Angle about z axis
- Θ = Pitch angle=Angle about y axis
- Φ =Roll angle=Angle about x axis
- P = x component of angular velocity
- Q = y component of angular velocity
- R = z component of angular velocity
- PN = x component of aircraft location in NED frame
- PE = y component of aircraft location in NED frame
- h = elevation about ground

The state vector of the system is:

X~ = [VT β α Ψ Θ Φ P Q R Pn Pe h]

And the state equations are:

Force Equations

$$\frac{dU}{dt} = R.V - Q.W - g0'.sin\theta + \frac{Fx}{m}$$

$$\frac{dV}{dt} = -R.U + P.W + g0'.sin\varphi cos\theta + \frac{Fy}{m}$$

$$\frac{dW}{dt} = Q.U - P.V + g0' cos\varphi cos\theta + \frac{Fz}{m}$$

Kinematic Equations

$$\frac{d\varphi}{dt} = P + tan\theta.(Qsin\varphi + Rcos\varphi)$$

$$\frac{d\theta}{dt} = Qcos\varphi - Rsin\varphi$$

$$\frac{d\psi}{dt} = \frac{Qsin\varphi + Rcos\varphi}{cos\theta}$$

Moment Equations

$$\frac{dP}{dt} = (c1.R + c2.P)Q + c3.L + c4.N$$

$$\frac{dQ}{dt} = c5.P.R - c6.(P^2 - R^2) + c7.M$$

$$\frac{dR}{dt} = (c8.P - c2.R)Q + c4.L + c9.M$$

Navigation Equations

$$\frac{dPn}{dt} = U.cos\theta cos\psi + V(-cos\varphi \cos\psi + sin\varphi sin\theta cos\psi) + W(sin\varphi \sin\psi + cos\varphi sin\theta cos\psi)$$

$$\frac{dPe}{dt} = Ucos\theta \sin\psi + V(cos\varphi \cos\psi + sin\varphi sin\theta \sin\psi) + W(-sin\varphi \cos\psi + cos\varphi sin\theta sin\psi$$

$$\frac{dh}{dt} = Usin\theta - Vsin\varphi cos\theta - wcos\varphi cos\theta)$$

In the above equations m represents the mass of aircraft, Fx,Fy,Fz are three components of acting force, and cis are constants depending on moment of inertia of the body about different axes.The above set of state equations actually represent the nonlinear model of an aircraft.

A simple longitudinal model for simulation :

This model has six degrees of freedom, it has fixed aerodynamic coefficients and its representative of a medium-sized transport aircraft at a low-speed flight condition. Augmentations are also provided for the effects of extending landing gear and flaps. The aircraft weighs 162,000 lb, and it has two turboprop engines each developing 30,000 lb of static thrust at sea level. The wing area is 2170 ft^2, wing span 140 ft, length 90 ft, cbar = 17.5 ft, pitch-axis inertia 4.1x10^6 slug-ft^2, and reference cg position 0.25cbar.

The mathematical model given here uses the wind tunnel data from NASA-Langley wind tunnel tests on a scale model of an F-16 airplane. This F-16 ,model has been programmed as a MATLAB function named f1. The code is shown below and the other helping functions are shown in the appendix. The quantities RM, XCGR and HE are respectively the reciprocal of the aircraft mass, x coordinate of the reference cg position and engine angular momentum. The equations are in body axes form with separate conversions to and from the variables Vt,α,β that have been used instead of U,V,W for the first three state variables. Subroutine ADC is used to calculate atmospheric density and hence dynamic pressure.

Cx(alpha,el) is a function subprogram that computes the non-dimensional force coefficient for the body x axis and is a function of angle of attack and elevator deflection. The total force coefficients for the three axes are CXT,CYT and CZT respectively.

Engine Model :

The NASA data include a model of the F-16 afterburning turbofan engine, in which the thrust response is modeled with a first-order lag, and the lag time constant is a function of the actual engine power level(POW) and the command power level(CPOW). This time constant is calculated in the function PDOT, whose value is the rate of change of power, while the state variable X13 represents the actual power level. The function TGEAR(throttle gearing) relates the commanded power to the throttle position(0 to 1.0). The variation of engine thrust with power level, altitude and MACH number is contained in the function THRUST.

Testing the Model :

Sign convention :

	Deflection	Sense	Effect
Elevator	Trailing edge down	Positive	Negative pitching momen
Rudder	Trailing edge left	Positive	Negative yawing moment,positive rolling moment
Ailerons	Right-wing trailing edge down	Positive	Negative rolling moment

Nominal Values:

Element	X(i)	dX(i)/dt
1	500	-75.23724
2	0.5	-0.8813491
3	-0.2	-0.4759990
4	-1	2.505734
5	1	0.3250820
6	-1	2.145926
7	0.7	12.62679
8	-0.8	0.9649671
9	0.9	0.5809759
10	1000	342.4439
11	900	-266.7707
12	10000	248.1241
13	90	-58.68999

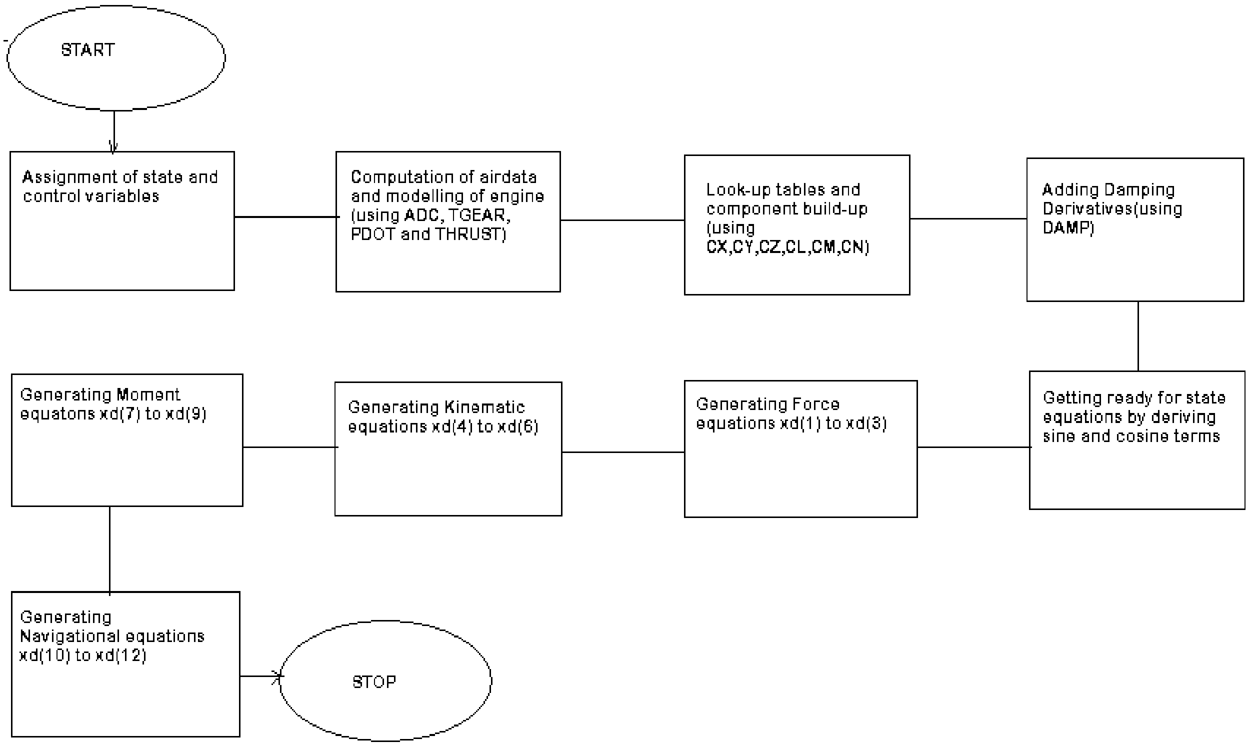
Now the MATLAB program along with the helping functions are given in flow chart form later and the plots of rate of change of a few variables wrt change of angle of attack and sideslip angle will also be shown later on.

```
% A six degree of freedom nonlinear aircraft model
function[xd]=f1(time,x,xd);
%-----assign------------------------------------------------------------
thtl=.77;
el=0.43;
ail=0.75;
rdr=0.15;
xcg=2.85;
qsphq=35000;
qsph=30000;
%----------------------------------------------------------------------
s=300; b=30; cbar=11.32; rm=1.57*10^-3; xcgr=0.35; he=160.0;
%----------------------------------------------------------------------
c1=-0.77; c2=0.02755; c3=1.055*10^-4; c4=1.642*10^-6; c5=0.9604;
c6=1.759*10^-2; c7=1.792*10^-5; c8=-0.7336; c9=1.587*10^-5;
%----------------------------------------------------------------------
rtod=57.29578; g=32.17;
%----------------------------------------------------------------------
% Assign state and control variables
vt=x(1); alpha=x(2)*rtod; beta=x(3)*rtod;
phi=x(4); theta=x(5); psi=x(6);
p=x(7); q=x(8); r=x(9); alt=x(12); pow=x(13);
%----------------------------------------------------------------------
% Air data computer and engine model
[amach,qbar]=adc(vt,alt);
cpow=tgear(thtl);
xd(13)=pdot(pow,cpow);
t=thrust(pow,alt,amach);
%----------------------------------------------------------------------
% Look up tables and component build up
cxt=cx(alpha,el);
```

```
cyt=cy(beta,ail,rdr);
czt=cz(alpha,beta,el);
dail=ail/20.0; drdr=rdr/30.0;
clt=cl(alpha,beta)+dlda(alpha,beta)*dail+dldr(alpha,beta)*drdr;
cmt=cm(alpha,el);
cnt=cn(alpha,beta)+dnda(alpha,beta)*dail+dndr(alpha,beta)*drdr;
%-------------------------------------------------------------------------
% Add damping derivatives
tvt=0.5/vt; b2v=b*tvt; cq=cbar*q*tvt;
d=damp(alpha);
cxt=cxt+cq*d(1);
cyt=cyt+b2v*(d(2)*r+d(3)*p);
czt=czt+cq*d(4);
clt=clt+b2v*(d(5)*r+d(6)*p);
cmt=cmt+cq*d(7)+czt*(xcgr-xcg);
cnt=cnt+b2v*(d(8)*r+d(9)*p)-cyt*(xcgr-xcg)*cbar/b;
%-------------------------------------------------------------------------
% Get ready for state equations
cbta=cos(x(3)); u=vt*cos(x(2))*cbta;
v=vt*sin(x(3)); w=vt*sin(x(2))*cbta;
sth=sin(theta); cth=cos(theta); sph=sin(phi);
cph=cos(phi); spsi=sin(psi); cpsi=cos(psi);
qs=qbar*s; qsb=qs*b; rmqs=rm*qs;
gcth=g*cth; qsphq*sph;
ay=rmqs*cyt; az=rmqs*czt;
%-------------------------------------------------------------------------
% Force equations
udot=r*v-q*w-g*sth+rm*(qs*cxt+t);
vdot=p*w-r*u+gcth*sph+ay;
wdot=q*u-p*v+gcth*cph+az;
dum=(u*u+w*w);
xd(1)=(u*udot+v*vdot+w*wdot)/vt;
xd(2)=(u*wdot-w*udot)/dum;
xd(3)=(vt*vdot-v*xd(1))*cbta/dum;
%-------------------------------------------------------------------------
% Kinematics
xd(4)=p+(sth/cth)*(qsph+r*cph);
xd(5)=q*cph-r*sph;
xd(6)=(qsph+r*cph)/cth;
%-------------------------------------------------------------------------
% Moments
xd(7)=(c2*p+c1*r+c4*he)*q+qsb*(c3*clt+c4*cnt);
xd(8)=(c5*p-c7*he)*r+c6*(r*r-p*p)+qs*cbar*c7*cmt;
xd(9)=(c8*p-c2*r+c9*he)*q+qsb*(c4*alt+c9*cnt);
%-------------------------------------------------------------------------
% Navigation
t1=sph*cpsi; t2=cph*sth; t3=sph*spsi;
s1=cth*cpsi; s2=cth*spsi; s3=t1*sth-cph*spsi;
s4=t3*sth+cph*cpsi; s5=sph*cth; s6=t2*cpsi+t3;
s7=t2*spsi-t1; s8=cph*cth;

xd(10)=u*s1+v*s3+w*s6;      % North Speed
xd(11)=u*s2+v*s4+w*s7;      % East speed
xd(12)=u*sth-v*s5-w*s8;     % Vertical Speed
%-------------------------------------------------------------------------
% Outputs
an=-az/g; alat=ay/g;
```

Flow chart:



Steady State Trimmed Flight:

We have obtained 12 sets of non-linear equations so far. The solution of these 12 non-linear equation gives the steady state flight condition. Obviously this solution cannot be obtained analytically because of very complex functional dependence of the aerodynamic data. Instead , we will do this by a numerical algorithm which iteratively adjusts the independent variables until some solution criterion is met.

Variable Specification:

All of the control variables (THTL,EL,AIL,RDR) enter the model only through tabular aerodynamic data, and we can't determine any analytical constraints on these control inputs. So, these must be adjusted by our numerical algorithm. Since only altitude component of NED position vector is relevant, so we can temporarily eliminate the three position states. First take the case of steady translational flight. The state variable φ,P,Q,R are all zero. Orientation ψ can be specified freely. This leaves Vt,α,β,θ to be considered. The sideslip angle(β) must be adjusted to zero. This leaves Vt,α, and θ, the first two are interrelated through amount of lift needed to balance the weight of aircraft. Instead of θ we will specify γ because we will use this as constraint. So finally we choose to specify Vt and γ. Now, consider the second case, steady state turning flight. Φ,P,Q,R can no longer be set to zero. The turn can be specified by Euler angle rate dψ/dt (rate at which aircraft heading changes). θ and φ can be fixed from ROC and Turn-coordination constraints. P,Q,R can be obtained from kinematic equation using θ,φ.

Rate of Climb(ROC) Constraint:

The rate of climb is simply Vtsinγ, and this is the negative z-component of velocity in NED frame. We will use coordinate transformation from wind axes to NED axes to obtain the constraints. So,

$$\begin{bmatrix} * \\ * \\ -Vtsin\gamma \end{bmatrix} = B\psi t.B\theta t.B\varphi t.St.\begin{bmatrix} Vt \\ 0 \\ 0 \end{bmatrix}$$

The asterisks indicate 'don't care's. If this equation is expanded and arranged to solve for θ,then

$$\sin\gamma = a\sin\theta - b\cos\theta$$

where $a = \cos\alpha\cos\beta$ & $b = \sin\varphi\sin\beta + \cos\varphi\sin\alpha\cos\beta$

Now solving for θ, we find:

$$tan\theta = \frac{ab + sin\gamma\sqrt{a^2 - \sin^2\gamma + b^2}}{a^2 - \sin^2\gamma}$$

Turn Co-ordination Constraint:

In NED frame the velocity vector is tangential to the turning circle ,so the centripetal acceleration in g's is:

$$G = \frac{Vt.\frac{d\psi}{dt}}{go'}$$

If we take the lateral equations of the nonlinear force equations and impose steady state condition dV/dt =0 and the coordination condition Fy=0, we obtain

$$0 = -R.U + P.W + g0'.\sin\varphi.\cos\theta$$

Rearranging the above equation we get:

$$tan\varphi = G.\frac{cos\beta}{cos\alpha}.\frac{(a-b^2)+b.tan\alpha.\sqrt{c(1-b^2)+G^2\sin^2\beta}}{a^2-b^2(1+c.\tan^2\alpha)}$$

Steady State Trimming Algorithm:

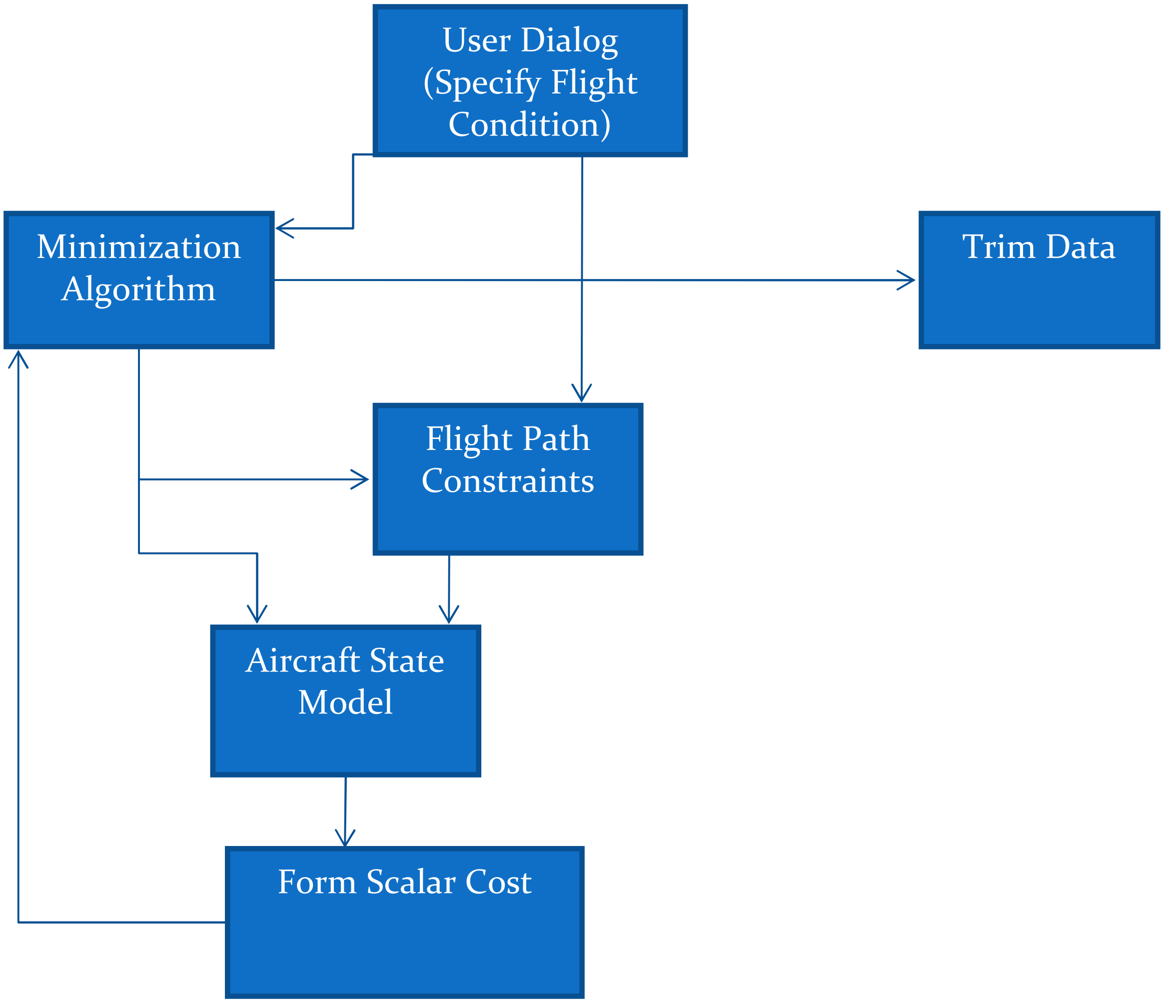
A convenient way to do this trim, with a readily available numerical algorithm, is to form a cost function from the sum of the squares of the derivatives of state variables.

The cost function used here is:

$$cost = xd(1)^2 + 100 * xd(2)^2 + xd(3)^2 + 10.xd(7)^2 + xd(8)^2 + xd(9)^2$$

A multivariable numerical optimization algorithm can then be used to adjust the control variables and appropriate state variables, to minimize the scalar cost. Example of such suitable algorithm is ‘fminsearch’ for multiple variable function minimization provided in MATLAB.

Trim Flow-chart:



Trim Results:

Here steady state level flight is considered.
The inputs which are to be given :

1. The altitude of flight
2. The speed of the aircraft

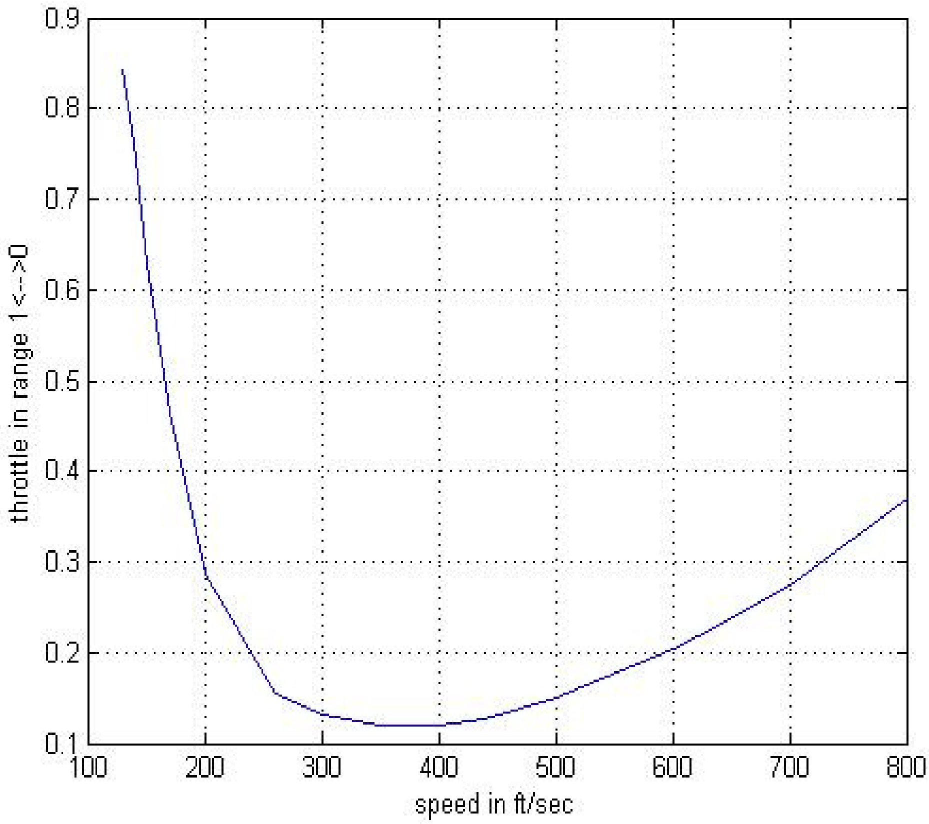
The trim program itself estimates the other state variables and control inputs. The minimization algorithm can minimize the cost function to 10^-22 order. After minimizing, the program plots the dynamics of aircraft for a given time limit.

All the following results are obtained at sea level i.e. at zero altitude.

Speed(ft/sec)	Throttle(0-1.0)	AOA(degree)	Elevator(degree)
130	.842	52.1	-10.47
140	.763	42.27	-14.49
150	.627	35.49	-11.85
170	.466	27.72	-8.13
200	.287	20.01	-6.08
260	.155	11.65	-4.02
300	.132	8.71	-3.78
350	.120	6.19	-2.86
400	.120	4.48	-2.31
440	.127	3.46	-2.02
500	.151	2.34	-1.72
540	.171	1.79	-1.57
600	.205	1.14	-1.38
640	.229	0.81	-1.28
700	.275	0.43	-1.21
800	.369	.005	-1.18

Plots:

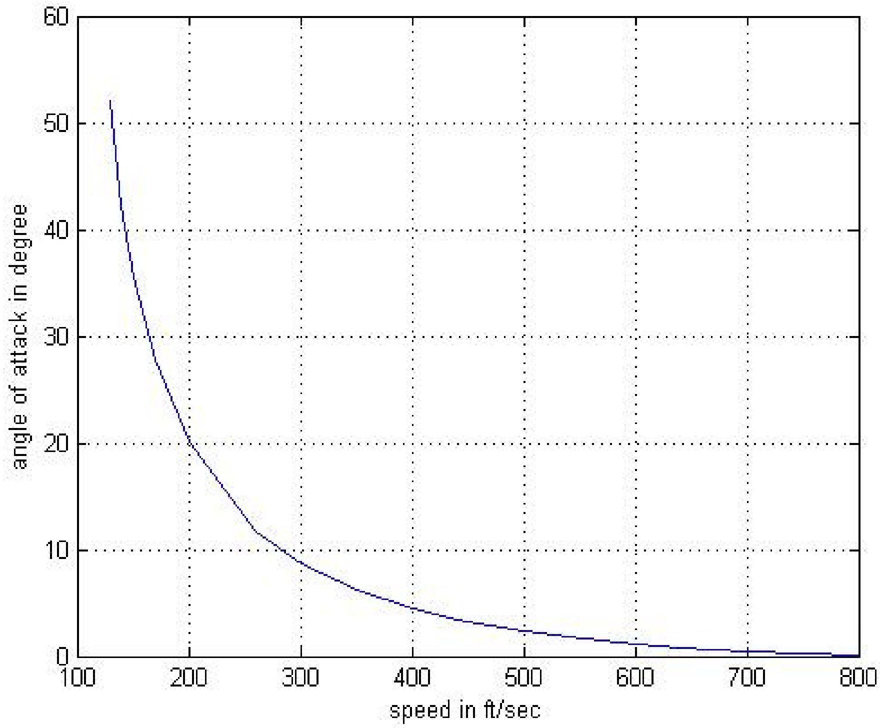
The throttle vs speed curve also known as Power Curve:



Inferences from Power Curve:

From the above Power Curve we can see a minimum throttle at a particular speed. So for a given altitude it is customary to run the aircraft at this speed to get minimum fuel consumption. The left-side of minimum is called the back-side of Power Curve and the right-side is called the front-side. When the aircraft is operated at back-side, there is no significant change in speed due to increase in throttle opening, the effect will be increase in altitude. And at front-side with increase in throttle opening speed will increase.

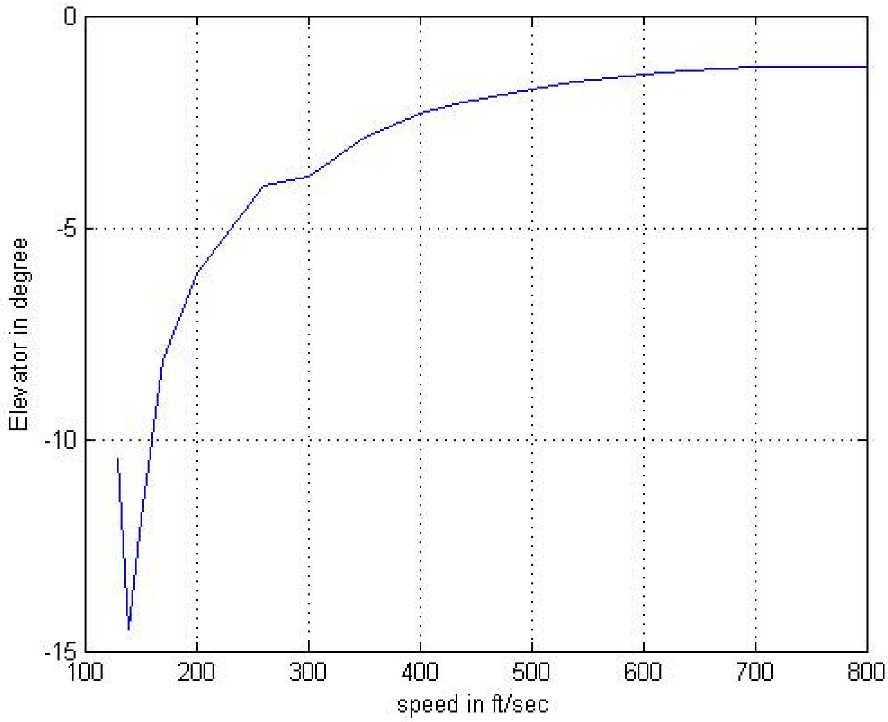
The angle of attack vs speed curve:



Important to note:

At high values of angle of attack, large drag will be generated. Wing sweep is used to take care of this. But it has a disadvantage that it also slopes down the lift-curve. A way to overcome this when a high lift to drag ratio is required is to use a variable sweep wing(F14 & B1B aircraft). But it is a very costly solution. For commercial aircrafts most common method of achieving high lift at low speed is to increase the camber of wing by means of leading and trailing edge devices(flaps and slats). A more specialized solution is to use an automatic maneuvering flap, as in the case of F16 leading edge flap which is deployed automatically as a function of angle of attack when the Mach number is low. In a typical trim example given later on the contribution of leading edge flap can be observed.

The elevator vs speed curve:

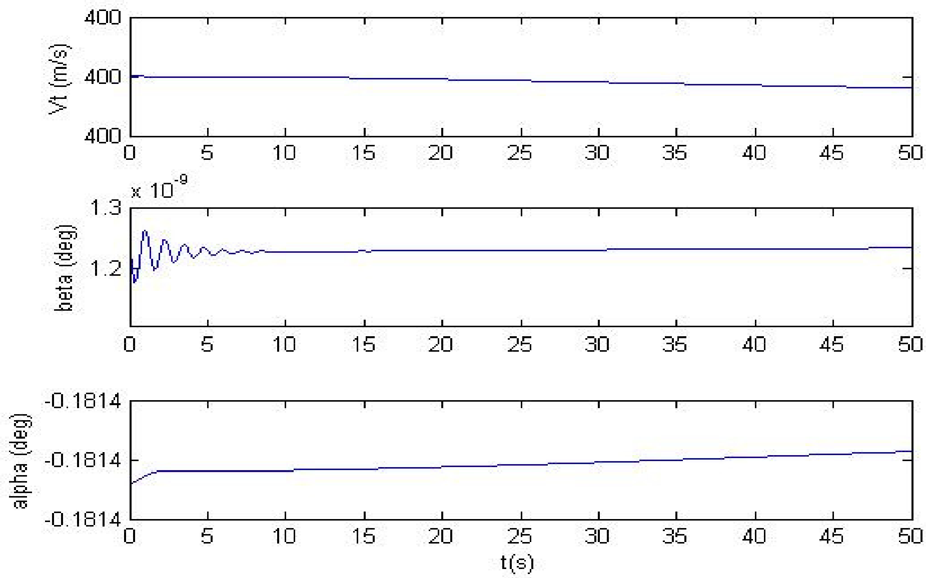


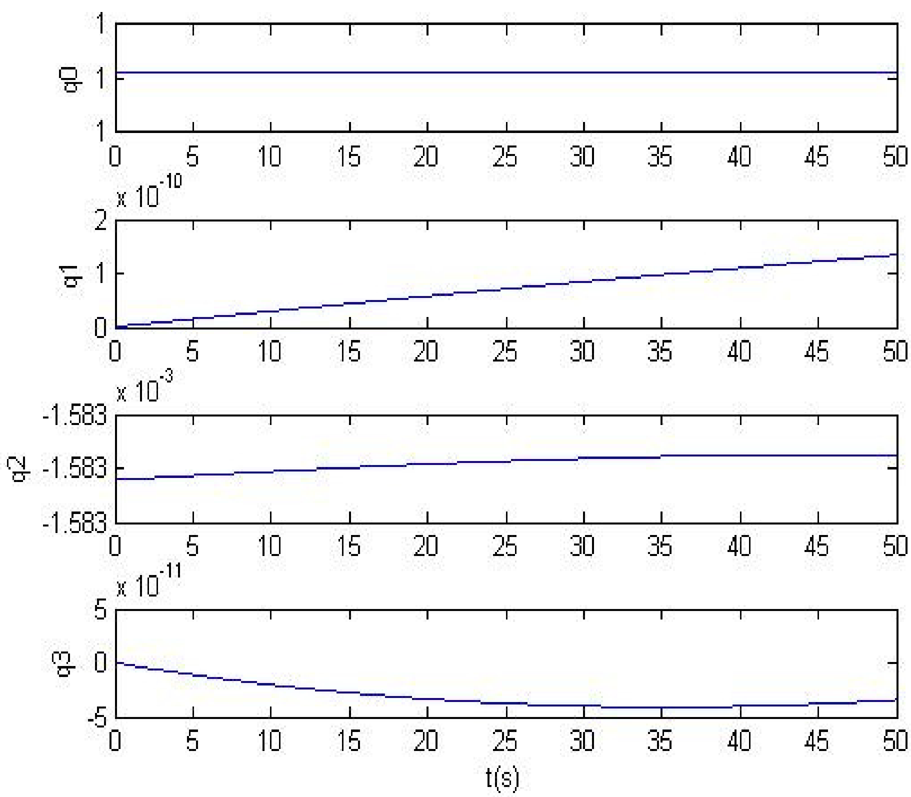
A typical complete trim output:

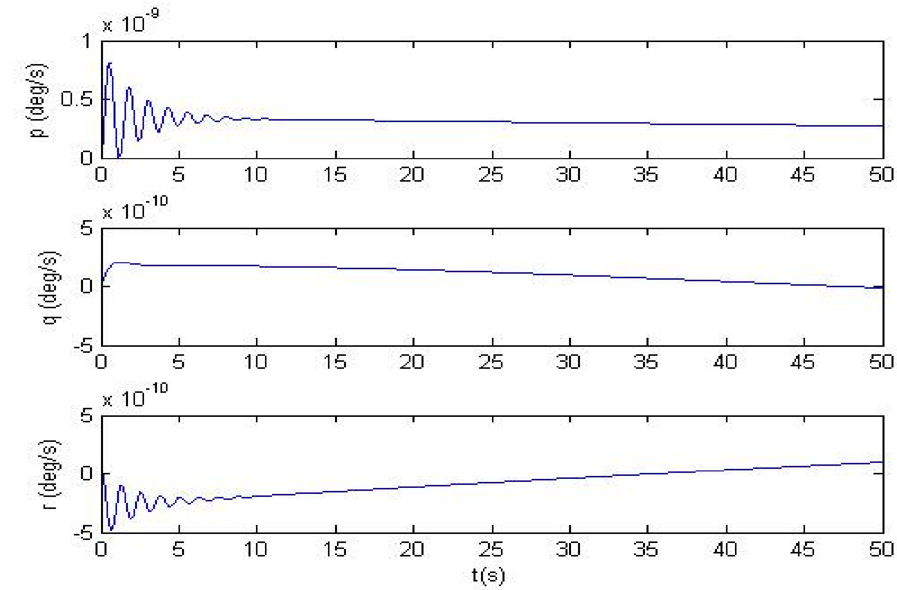
At altitude = 20000 ft(6096 m)
& speed = 400ft/sec (121.92 m/sec)
The trimmed flight condition is:

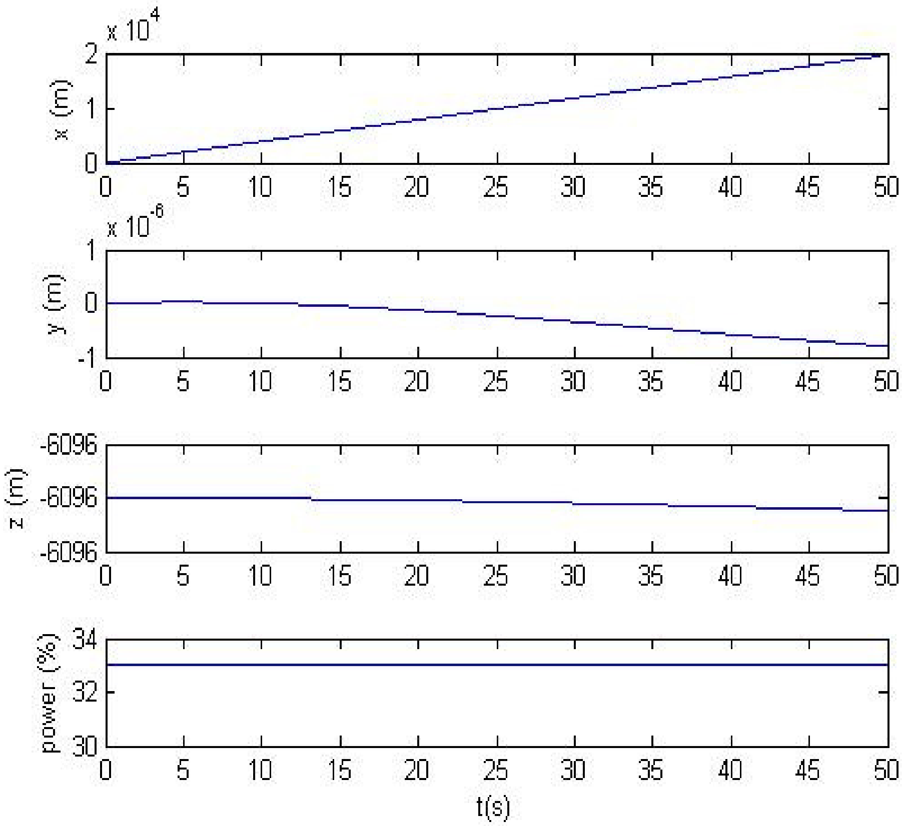
cost = 1.2797e-022	Vel. = 121.92 m/s
dth = 0.31904 -	pow = 20.7186 %
elev = -3.9689 deg	
ail = 1.7859e-009 deg	
rud = -5.3951e-009 deg	
alpha = 11.342 deg	
dLEF = 16.2888 deg	

Now if the trimmed control inputs are kept intact the flight dynamics for next 50 seconds will be following:









In some of the above dynamic responses a little bit of oscillations may be seen. But the value of the state variable in these cases are negligible. By virtue of this they can be considered as steady state conditions.

References:

- Brian L. Stevens, Frank L. Lewis , ” Aircraft Control and Simulation ”, pp : 1-41 , 1992.
- http://en.wikipedia.org/wiki/Flight_dynamics

Conclusion:

Trimming calculations are done using the 6-DOF nonlinear aircraft model. To develop this model extensive data and algorithms are taken from the book of Stevens. In the trimming algorithm the book used Simplex algorithm for cost minimization. It can minimize the cost function to 10^-12 order. Instead of doing so, here nonlinear minimization function ‘fminsearch’ , provided in MATLAB, is used. It can minimize the cost function to 10^-23 order more efficiently. So the steady state trimmed condition obtained here is more acceptable. The Power Curve obtained here perfectly matches with the same given in the book.